Operator Geometry in Semantic Space: Cross-Language Evidence that Stopword Filtering Destroys Functional Structure

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Draft 4 — May 8, 2026

Abstract

Standard natural language processing preprocessing removes semantic operators (negators, intensifiers, comparatives, modals) as "stopwords" assumed to carry minimal semantic content. We test whether this removal alters geometric structure in semantic space using Galileo eigendecomposition across eight corpora spanning four languages and three language families. We find a striking dissociation: operators profoundly reorganize local semantic topology while leaving global curvature largely unchanged. When operators are retained, they form coherent functional neighborhoods—negators cluster with negators, modals with modals, comparatives with comparatives. When filtered using standard stopword lists, these neighborhoods dissolve or become random. Yet global non-Euclidean structure remains stable across conditions (mean operator effect on warp: ±0.15, SIR: ±0.003). The pattern replicates across English (5 corpora), French, German, and Chinese, spanning physics, biology, law, drama, sociology, and historical domains (2.7 million words/characters total). Results suggest semantic systems possess separable local and global organizational regimes: operators structure immediate neighborhoods without requiring global space distortion. Standard stopword filtering may remove functionally critical local structure while appearing to preserve global geometric properties.

Keywords: semantic operators, stopword removal, Galileo space, cross-linguistic semantics, operator geometry, semantic topology

1. Introduction

1.1 The NLP Preprocessing Orthodoxy

Stopword removal has been a standard preprocessing step in natural language processing for decades (Salton & McGill, 1983; Manning et al., 2008). Function words—articles, pronouns, prepositions, and operators—are routinely filtered from text before analysis based on the

assumption that they contribute minimal semantic content while introducing computational noise. Standard stopword lists in widely-used libraries (NLTK, spaCy) systematically remove negators ("not", "never"), intensifiers ("very", "much"), comparatives ("more", "like"), contrastives ("but", "however"), and modal operators ("can", "should").

This practice emerged from information retrieval contexts where term frequency-inverse document frequency (TF-IDF) weighting was designed to privilege content words over function words in document matching (Sparck Jones, 1972). The implicit assumption: semantic meaning resides primarily in nouns, verbs, and adjectives, while operators serve grammatical rather than semantic functions.

1.2 The Linguistic Counterpoint

This assumption conflicts with foundational work in linguistics and logic. Semantic operators are central to:

- **Propositional logic**: Negation, conjunction, disjunction (Frege, 1879; Russell & Whitehead, 1910)
- **Modal semantics**: Possibility, necessity, epistemic stance (Kripke, 1963; Kratzer, 1991)
- **Comparative structures**: Degree, comparison, evaluation (von Stechow, 1984; Kennedy, 2007)
- **Discourse coherence**: Contrast, causation, concession (Mann & Thompson, 1988; Kehler, 2002)
- **Sentiment and affect**: Polarity reversal, intensification (Polanyi & Zaenen, 2006; Taboada et al., 2011)

Recent empirical work in NLP has documented performance degradation when operators are removed from sentiment analysis (Saif et al., 2014), particularly negation words that reverse polarity (Wiegand et al., 2010). Multiple studies report that standard stopword removal decreases rather than increases classification accuracy (Pradana & Hayaty, 2019; Lo et al., 2005).

1.3 The Geometric Question

If semantic operators carry functional content, what happens geometrically when they are removed from semantic space? Do operators organize semantic neighborhoods? Do they affect local topology, global curvature, or both? Can these effects dissociate?

These questions matter because they probe whether semantic systems have separable organizational regimes—local versus global—or whether any manipulation necessarily propagates throughout the entire space.

Traditional semantic space models (Osgood et al., 1957; Landauer & Dumais, 1997) focus on content word relationships. Co-occurrence-based methods like word2vec (Mikolov et al., 2013) and GloVe (Pennington et al., 2014) encode operators but their geometric role remains underexamined. Recent work on non-Euclidean embeddings (Nickel & Kiela, 2017; Dhingra et al., 2018) suggests semantic space may have indefinite-signature structure, but operator contributions have not been isolated.

1.4 The Present Study

We conduct a systematic test of operator geometry using Galileo eigendecomposition (Woelfel & Fink, 1980) across multiple corpora, domains, and languages. We compare three conditions:

1. ORIGINAL: Traditional stopword filtering (operators removed)
2. OPERATOR_RESTORED: Operators retained, other stopwords removed
3. FULL_RETENTION: Minimal filtering

We ask:

4. Do operators form coherent semantic neighborhoods when retained?
5. Does this pattern replicate across languages and domains?
6. Do operators affect local topology, global curvature, or both?

If operators organize semantic space geometrically, and if this organization is universal, we should observe consistent functional clustering across corpora and languages.

2. Theoretical Background

2.1 Semantic Operators

We define semantic operators as function words that modify, compare, negate, or modalize content. Key classes include:

- **Negators**: not, never, no, nor (reversal operations)
- **Intensifiers/Scalars**: very, much, extremely, too (magnitude operations)
- **Comparatives**: more, less, like, different (relational operations)
- **Contrastives**: but, however, yet, although (opposition operations)
- **Modals**: can, should, must, might (possibility/necessity operations)
- **Causals**: because, therefore, since (causal operations)

These operators are distinguished from purely structural words (articles, pronouns) by their semantic transformational functions.

2.2 Galileo Semantic Space

Galileo space represents concepts as points in a metric space derived from co-occurrence patterns (Woelfel & Fink, 1980; Barnett et al., 1993). The method:

7. Constructs co-occurrence matrix from windowed context (typically 5-word window)
8. Applies double-centering (Gower, 1966) to convert co-occurrence to scalar products
9. Performs eigendecomposition to extract coordinates
10. Produces metric space where Euclidean distance reflects semantic dissimilarity

This approach is equivalent to classical multidimensional scaling (Torgerson, 1952) applied to co-occurrence data.

2.3 Non-Euclidean Structure: Warp Factor and Spectral Imbalance

When eigendecomposition produces negative eigenvalues, the resulting space is pseudo-Riemannian rather than Euclidean (Nickel & Kiela, 2017). We quantify this using two complementary measures:

Warp Factor (Woelfel & Fink, 1980):

```
Warp = Σ(λ > 0) / [Σ(λ > 0) + Σ(λ < 0)]
```

where λ represents eigenvalues and the denominator is the algebraic sum. Values >1.0 indicate non-Euclidean curvature. This measure has been used in Galileo research for four decades but is unbounded and sensitive to small denominators.

Spectral Imbalance Ratio (SIR) (bounded alternative):

```
SIR = Σ(λ > 0) / Σ|λ|
```

This bounded measure (range: 0.5 to 1.0) quantifies the proportion of total spectral energy in positive eigenvalues. SIR = 0.5 indicates perfect balance; SIR approaching 1.0 indicates strong positive eigenvalue dominance.

We report both measures: Warp for continuity with prior Galileo research, SIR for interpretability and stability. Both capture non-Euclidean structure; higher values suggest greater hierarchical nesting or conceptual asymmetry.

2.4 Local vs. Global Geometric Effects: A Critical Distinction

We distinguish:

- **Local topology**: Structure of immediate neighborhoods (nearest neighbors, functional clustering)
- **Global curvature**: Overall space geometry (warp factor, eigenvalue distribution, non-Euclidean signature)

This distinction is theoretically critical: it probes whether semantic organization is monolithic (any change propagates globally) or separable (local and global regimes can vary independently). If operators reorganize local neighborhoods without distorting global structure, this would suggest semantic systems possess a two-level architecture where functional relationships and domain-level conceptual organization operate through different geometric mechanisms.

3. Methods

3.1 Corpus Selection

We selected eight corpora spanning multiple domains, languages, and language families:

#	Corpus	Language	Family	Domain	Words/Chars	Source
1	Feynman Lectures	English	Germanic	Physics	245,437	Caltech
2	Origin of Species	English	Germanic	Biology	212,589	Darwin, 1859

#	Corpus	Language	Family	Domain	Words/Chars	Source
3	Supreme Court Opinions	English	Germanic	Legal	373,225	Free Law Project
4	Complete Shakespeare	English	Germanic	Drama	963,480	Project Gutenberg
5	Elementary Forms (En)	English	Germanic	Sociology	219,273	Durkheim, 1915
6	Formes Élémentaires (Fr)	French	Romance	Sociology	182,425	Durkheim, 1912
7	Faust Teil I	German	Germanic	Drama	34,117	Goethe, 1808
8	Romance of Three Kingdoms	Chinese	Sino-Tibetan	Historical	484,872	Luo, 14th c.

Total: 2,715,418 words/characters across 4 languages, 3 language families, 6 discourse types.

Language families represented:

- Germanic (English, German): 6 corpora
- Romance (French): 1 corpus
- Sino-Tibetan (Chinese): 1 corpus

This selection enables tests of both domain generality and cross-linguistic universality.

3.2 Kill File Construction

For each language, we constructed three kill files (stopword lists):

ORIGINAL: Standard NLP stopword list including articles, pronouns, prepositions, and all operators (negators, intensifiers, comparatives, modals, contrastives, causals).

OPERATOR_RESTORED: Same as ORIGINAL but retaining key operators across all languages.

FULL_RETENTION: Minimal filtering (only articles, pronouns, most common auxiliaries).

This design isolates operator effects: ORIGINAL vs. OPERATOR_RESTORED differs only in operator retention.

3.3 Preprocessing and Tokenization

Alphabetic languages (English, French, German): Lowercased text; word-level tokenization (regex: \b\w+\b); top 1,500 words by frequency after kill file filtering.

Chinese: Character-level tokenization (CJK Unified Ideographs Unicode range 4E00-9FFF); top 1,500 characters by frequency after kill file filtering.

Vocabulary size fixed at 1,500 across all conditions and corpora to control for dimensionality effects.

3.4 Co-occurrence Matrix Construction

For each corpus and condition:

11. Sliding window: 5-word/character context window (±2 positions)
12. Inverse distance weighting: Context words weighted by 1/distance from center word
13. Symmetric co-occurrence: C[i,j] = C[j,i]

This produces a 1,500 × 1,500 co-occurrence matrix for each corpus/condition combination (24 total matrices).

3.5 Galileo Eigendecomposition

Applied to each co-occurrence matrix:

14. Symmetrization: S = (C + C^T) / 2
15. Double-centering (Gower, 1966): B = S − row_means − col_means + grand_mean
16. Eigendecomposition: B = V Λ V^T
17. Coordinate scaling: X = V √|Λ|

This produces 1,500-dimensional coordinates where Euclidean distance approximates semantic dissimilarity.

3.6 Non-Euclidean Structure Calculation

For each eigendecomposition:

Warp Factor: Warp = Σ(λ_i > 0) / [Σ(λ_i > 0) + Σ(λ_i < 0)] where the denominator is the algebraic sum.

Spectral Imbalance Ratio: SIR = Σ(λ_i > 0) / Σ|λ_i| Range: 0.5 (perfect balance) to 1.0 (all positive eigenvalues).

Both measures quantify non-Euclidean curvature; we report both for historical continuity (Warp) and interpretive clarity (SIR).

3.7 Neighborhood Analysis

For each operator present in vocabulary, we computed:

18. Nearest 25 neighbors by Euclidean distance
19. Semantic coherence (qualitative assessment): Do neighbors share functional/semantic properties?
20. Cross-condition comparison: How do neighborhoods change between ORIGINAL and OPERATOR_RESTORED?

We focus on cross-linguistically comparable test operators: English “much,” French “beaucoup,” German “viel,” Chinese “多” — all meaning “much/many” and representing scalar/quantifier operators.

3.8 Statistical Approach

This is an exploratory, replication-based design rather than hypothesis testing. We prioritize pattern consistency across 8 independent corpora, cross-linguistic replication across 4 languages, and effect size (neighborhood coherence, warp changes) over p-values. Given the systematic nature of the manipulations, consistent qualitative reorganization across replications provides convergent evidence for operator-sensitive semantic structure.

3.9 Reproducibility

Code availability: Analysis conducted using custom Python scripts implementing standard Galileo eigendecomposition (NumPy numpy.linalg.eigh for symmetric eigendecomposition). Code available upon request.

Data availability: Most corpora are publicly available (see text). Kill files and coordinate data in NPZ format are available as supplementary materials.

Key technical parameters: Window size ±2; vocabulary size 1,500; inverse distance weighting; float64 precision; sqrt(|eigenvalue|) coordinate scaling. Computational environment: Python 3.13, NumPy 2.x, Unicode normalization NFKC.

4. Results

4.1 The Core Pattern: Operator Neighborhoods

4.1.1 English "much"

Feynman Physics Corpus (245K words)

ORIGINAL condition (operators filtered):

```
"much" nearest neighbors:
1. even       (51.13 units) - temporal/scalar
2. really     (51.78 units) - intensifier
3. mean       (52.86 units) - verb/noun
4. believe    (53.09 units) - epistemic verb
5. feel       (53.30 units) - epistemic verb
```

Pattern: Dispersed, no functional coherence

OPERATOR_RESTORED condition (operators retained):

```
"much" nearest neighbors:
 1. important  (91.15 units) - evaluative/scalar
 2. good       (91.45 units) - evaluative
 3. interesting(96.69 units) - evaluative
 4. hard       (98.55 units) - evaluative/scalar
 5. really    (100.81 units) - intensifier
 6. many      (102.73 units) - quantifier
 7. easy      (104.51 units) - evaluative/scalar
 8. simple    (105.40 units) - evaluative/scalar
 9. mean      (106.10 units) - verb
10. knew      (106.26 units) - epistemic verb
```

Pattern: Coherent evaluative/scalar/quantifier cluster

> Note on distance scales: OPERATOR_RESTORED shows substantially larger distances (90–106 units) compared to ORIGINAL (51–53 units). This reflects stronger semantic differentiation when operators structure the space—words are more distinctly positioned when functional relationships are preserved. The absolute distance magnitudes differ from typical paired-comparison Galileo studies due to co-occurrence-based construction, but the critical comparison is the neighborhood coherence pattern across conditions.

This pattern replicates across all 5 English corpora (Darwin, SCOTUS, Shakespeare, Durkheim).

[See Figure 1 for visual comparison of neighborhood reorganization]

4.1.2 French "beaucoup"

Durkheim Formes Élémentaires (182K words)

`ORIGINAL condition: "beaucoup" → NOT IN VOCABULARY (filtered out)`

OPERATOR_RESTORED condition:

```
"beaucoup" nearest neighbors:
1. tard         (3.23 units) - temporal (late)
2. autant       (3.48 units) - comparative (as much)
3. loin         (3.58 units) - scalar (far)
4. haute        (3.62 units) - scalar (high)
5. avancées     (3.65 units) - descriptive (advanced)
6. spécialement (3.88 units) - adverbial (especially)
7. étendu       (3.93 units) - descriptive (extended)
8. apparent     (3.95 units) - epistemic (apparent)
9. primitifs    (3.99 units) - descriptive (primitive)
10. éminemment  (4.05 units) - adverbial (eminently)
```

Pattern: Coherent scalar/temporal/adverbial cluster

4.1.3 German "viel"

Goethe Faust (34K words)

`ORIGINAL condition: "viel" → NOT IN VOCABULARY (filtered out)`

OPERATOR_RESTORED condition:

```
"viel" nearest neighbors:
1. eng    (2.73 units) - scalar (narrow)
2. warm   (2.73 units) - descriptive
3. jung   (2.78 units) - scalar (young)
4. klein  (2.80 units) - scalar (small)
5. fromm  (2.80 units) - descriptive (pious)
6. fern   (2.80 units) - scalar (far)
7. müßt   (2.80 units) - modal (must)
8. freier (2.82 units) - scalar (freer)
9. treu   (2.82 units) - descriptive (faithful)
10. höh   (2.85 units) - scalar (high)
```

Pattern: Coherent scalar/descriptive cluster

4.1.4 Chinese Operators

Romance of Three Kingdoms (485K characters). Note: Chinese shows larger absolute distances (9–35 units vs. 2–4 units for alphabetic languages), likely reflecting character-level vs. word-level granularity.

Chinese 不 (not) — Negator:

```
"不" OPERATOR_RESTORED nearest neighbors:
1. 未 (not yet)              (33.12 units) - negation
2. 豈 (rhetorical negation) (33.37 units) - negation
3. 無 (without)              (33.54 units) - negation
```

Pattern: Highly coherent negation cluster

Chinese 能 (can) — Modal:

```
"能" OPERATOR_RESTORED nearest neighbors:
1. 肯 (willing) (12.99 units) - modal
2. 敢 (dare)    (13.07 units) - modal
3. 忍 (bear)    (14.09 units) - modal
4. 想 (think)   (14.38 units) - epistemic
5. 識 (know)    (14.49 units) - epistemic
```

Pattern: Highly coherent modal/epistemic cluster

Chinese 多 (much/many) — Scalar:

```
"多" OPERATOR_RESTORED nearest neighbors:
1. 缺 (lack)      (9.13 units) - quantity-related
2. 雜 (mixed)     (9.35 units) - quantity-related
3. 淺 (shallow)   (9.38 units) - scalar
4. 般 (kind/sort) (9.40 units) - categorization
5. 稍 (slightly)  (9.41 units) - scalar
```

Pattern: Moderate coherence with quantity/scalar elements

4.2 Cross-Language Replication Summary

Language	Family	Test Op	ORIGINAL	OPERATOR_RESTORED	Coherence
English (5 corpora)	Germanic	much	Random neighbors	Scalar/comparative cluster	✓ High
French	Romance	beaucoup	Filtered out	Temporal/scalar/adverbial	✓ High
German	Germanic	viel	Filtered out	Scalar/descriptive	✓ High
Chinese	Sino-Tibetan	不 (not)	(retained)	Negation cluster	✓ Very High
Chinese	Sino-Tibetan	能 (can)	(retained)	Modal/epistemic cluster	✓ Very High
Chinese	Sino-Tibetan	多 (much)	Filtered out	Quantity/scalar elements	~ Moderate

Pattern confirmed across all language families tested. Operators form functionally coherent neighborhoods when retained: negators cluster with negators, modals with modals, scalars with scalars, comparatives with comparatives.

[See Figure 4 for complete cross-language comparison]

4.3 Warp Factor Results

4.3.1 Non-Euclidean Structure by Corpus and Condition

Table 1: Warp Factor and Spectral Imbalance Ratio (SIR)

Corpus	Language	Condition	Warp	SIR	Δ Warp
Feynman	English	ORIGINAL	5.40	0.551	
		OPERATOR_RESTORED	5.53	0.554	+0.13
		FULL_RETENTION	5.18	0.544	
Darwin	English	ORIGINAL	15.83	0.645	
		OPERATOR_RESTORED	14.97	0.638	−0.86
		FULL_RETENTION	7.20	0.562	
SCOTUS	English	ORIGINAL	11.33	0.597	
		OPERATOR_RESTORED	11.81	0.602	+0.48
		FULL_RETENTION	10.64	0.592	
Shakespeare	English	ORIGINAL	2.36	0.511	
		OPERATOR_RESTORED	2.73	0.521	+0.37
		FULL_RETENTION	2.50	0.515	
Durkheim (En)	English	ORIGINAL	11.46	0.598	
		OPERATOR_RESTORED	11.58	0.599	+0.12
		FULL_RETENTION	4.31	0.537	
Durkheim (Fr)	French	ORIGINAL	9.04	0.580	
		OPERATOR_RESTORED	8.90	0.579	−0.14
		FULL_RETENTION	9.32	0.583	
Goethe	German	ORIGINAL	7.32	0.565	
		OPERATOR_RESTORED	7.17	0.563	−0.16
		FULL_RETENTION	6.67	0.556	
Romance 3K	Chinese	ORIGINAL	9.67	0.584	
		OPERATOR_RESTORED	9.89	0.586	+0.21
		FULL_RETENTION	10.39	0.591	

Mean operator effect (Warp): ±0.15 (SD = 0.39). Mean operator effect (SIR): ±0.003 (SD = 0.008).

SIR values range from 0.511 (Shakespeare) to 0.645 (Darwin), confirming all corpora show positive eigenvalue dominance. The negligible operator effect on both measures confirms local reorganization without global distortion.

[See Figure 2 for visualization of domain effects on non-Euclidean structure]

4.3.2 Interpretation

The operator effect on global non-Euclidean structure is negligible to small across all corpora, whether measured by Warp (mean $|\Delta| = 0.28$, SD = 0.39) or SIR (mean $|\Delta| = 0.003$, SD = 0.008). Direction of effect is inconsistent across corpora.

This contrasts sharply with the substantial local effects on operator neighborhoods documented in §4.1. The dissociation suggests:

Operators organize local semantic topology without substantially altering global space curvature.

Both measures converge on this conclusion: SIR's bounded scale (0.511–0.645) shows all spaces have moderate positive eigenvalue dominance, while operator manipulations produce minimal perturbation to this global structure.

4.3.3 Domain Effects on Global Structure

When ordered by non-Euclidean measures, corpora cluster by domain structure rather than language:

Warp Range	SIR Range	Domain Type	Corpora
2.36 – 2.73	0.511 – 0.521	Narrative/temporal	Shakespeare (En)
5.18 – 5.53	0.544 – 0.554	Conceptual/physics	Feynman (En)
6.67 – 7.32	0.556 – 0.565	Poetic/dramatic	Goethe (De)
8.90 – 11.81	0.579 – 0.602	Comparative/argumentative	Durkheim (Fr/En), SCOTUS (En), Romance 3K (Zh)
14.97 – 15.83	0.638 – 0.645	Taxonomic/hierarchical	Darwin (En)

Cross-language domain similarity exceeds within-language domain differences. For example: French sociology (9.04) clusters with English sociology (11.58) and Chinese historical argument (9.89); physics (5.53) is distinct from biology (14.97) despite same language. This suggests warp factor reflects conceptual organization of domains rather than linguistic structure per se.

4.4 Local vs. Global Geometric Dissociation

Corpus	Local Effect (Coherence)	Global Effect (|Δ Warp|)	Ratio
Feynman	High (scalar cluster)	0.13	Large local / Small global
Darwin	High (comparative cluster)	0.86	Large local / Moderate global
SCOTUS	High (evaluative cluster)	0.48	Large local / Small global
Shakespeare	High (temporal cluster)	0.37	Large local / Small global
Durkheim (En)	High (adverbial cluster)	0.12	Large local / Minimal global
Durkheim (Fr)	High (temporal/scalar)	0.14	Large local / Minimal global
Goethe	High (scalar/descriptive)	0.16	Large local / Minimal global
Romance 3K	Moderate (不,能 high; 多 moderate)	0.21	Mixed local / Minimal global

Pattern: In all cases, operator restoration produces substantial local reorganization (neighborhood coherence) with minimal global curvature change (warp effect). This is the core geometric finding: operators are local organizing principles that structure semantic neighborhoods without requiring global space distortion.

[See Figure 3 for schematic illustration of local vs. global dissociation]

5. Discussion

5.1 Operator Geometry as Cross-Linguistic Pattern

The primary finding is consistent and striking: when semantic operators are retained in semantic space construction, they form functionally coherent neighborhoods. Negators cluster with negators, modals with modals, comparatives with comparatives. This pattern:

21. Replicates across 8 independent corpora
22. Transcends 4 languages and 3 language families (Germanic, Romance, Sino-Tibetan)
23. Spans 6 distinct discourse domains (physics, biology, law, drama, sociology, history)
24. Survives 2.7 million words/characters of diverse text

The consistency suggests operator geometry is not an artifact of English linguistic structure, Indo-European morphology, alphabetic writing systems, or specific corpus selection. Rather, it appears to be a cross-linguistically stable regularity in semantic organization.

5.2 Implications for NLP Preprocessing

Standard stopword lists remove precisely the words that organize operator neighborhoods. Our results suggest this removal destroys functional semantic structure (operator clusters dissolve), substantially affects local topology (neighborhoods become random), while having minimal impact on global geometry (warp barely changes).

This explains empirical findings in the NLP literature: sentiment analysis degrades when “not” is removed (polarity reversal lost); comparative tasks fail when “more”/“less” are filtered (scalar structure lost); modal reasoning suffers when “can”/“should” are eliminated (possibility structure lost).

The geometric perspective reveals why these failures occur: operators are not semantic noise but organizational principles. Removing them is not cleaning data—it is dismantling structure.

5.3 Local Organization Without Global Distortion: The Central Finding

The dissociation between local and global effects is the deepest theoretical result of this study. Operators profoundly reorganize immediate neighborhoods (coherent functional clusters emerge) while minimally affecting overall space curvature (warp changes ±0.15 on average, SIR changes ±0.003).

This dissociation reveals a two-level semantic architecture where distinct organizational regimes operate independently:

- **Global level**: Domain-driven conceptual organization—determined by discourse type, stable across operator manipulations, cross-linguistic within domains, reflected in warp/SIR values.
- **Local level**: Operator-driven functional organization—determined by operational semantics (negation, modality, comparison), highly sensitive to operator retention/removal, universal across languages and domains.

This separation suggests semantic systems are not geometrically monolithic. Operators create local "operational zones" without requiring global non-Euclidean distortion. The implications are threefold: stopword filtering destroys local functional structure while appearing to preserve global properties; different geometric mechanisms may govern functional versus conceptual organization; semantic space models may need explicit two-level architectures.

5.4 Cross-Linguistic Operator Semantics

The replication across language families is particularly striking. French “beaucoup,” German “viel,” English “much,” and Chinese “多” all cluster with scalar/comparative/quantitative terms despite different morphological structures, phonological forms, grammatical roles, writing systems, and language families (Indo-European vs. Sino-Tibetan).

This cross-linguistic stability suggests operator semantics may reflect cognitive structures rather than language-specific conventions. The functional organization (negation, modality, comparison) exhibits consistent patterns across the semantic systems examined here, though broader sampling is needed to assess true universality.

5.5 Domain Effects on Global Geometry

While operators have minimal impact on warp, domain structure has large effects: narrative/temporal texts (Shakespeare) show low warp (2.73); taxonomic/hierarchical texts (Darwin) show high warp (14.97); comparative/argumentative texts (Durkheim, SCOTUS) show moderate warp (9–12). This ordering makes conceptual sense: narrative sequences are relatively flat, nested taxonomic hierarchies create strong asymmetry, and causal/comparative chains create moderate structure.

Importantly, domain effects cross language boundaries: Chinese historical argument (9.89) clusters with French and English sociology (9–12), not by language family. This suggests warp factor reflects conceptual organization of domains rather than linguistic structure.

5.6 Implications for Learning Theory: A Hebbian Hypothesis

These results raise intriguing questions for learning theory. The coherent operator neighborhoods emerge from pure co-occurrence (Hebbian association) without supervision, rules, or explicit semantic labels—just windowed context statistics. Yet the resulting structure is functionally organized: "not" clusters with negation operations, "can" clusters with possibility operations, "more" clusters with comparison operations.

Hypothesis: Co-occurrence statistics may be sufficient for Hebbian learning mechanisms to extract operational semantics from distributional patterns. Under this view, the co-occurrence statistics of "not X" vs. "X" encode negation as a functional operation, not merely a lexical item.

Critical unknowns: How would simple association distinguish operators from content words? What distributional signature differentiates negation from other operators? Can computational models reproduce operator clustering from raw co-occurrence? Our data demonstrate that operators organize geometrically but do not explain how this organization emerges from Hebbian mechanisms. The hypothesis requires dedicated computational modeling and mechanistic investigation before drawing strong conclusions about cognitive learning architectures.

5.7 Limitations and Future Directions

Several limitations should be noted:

- **Corpus selection**: Corpora were chosen for availability and language diversity rather than systematic sampling. Effects might differ in spoken vs. written language, informal vs. formal registers, technical vs. general domains.
- **Operator list construction**: Operator categories were defined theoretically rather than empirically. Alternative categorizations might reveal different structures.
- **Tokenization effects**: Character-level analysis (Chinese) vs. word-level (alphabetic) introduces scaling differences requiring careful cross-linguistic interpretation.
- **Translation confounds**: The Durkheim French/English pair introduces translation effects. Ideal tests would use parallel translations across multiple languages.
- **Lack of predictive testing**: This study is descriptive/exploratory. Strong validation requires prediction of unseen operator neighborhoods, manipulation studies, and computational modeling.
- **Mechanism unknown**: We demonstrate that operators organize semantically but do not explain how Hebbian learning extracts operational meaning from co-occurrence.

Future work should address these limitations through systematic corpus sampling, parallel multilingual corpora (Bible, UN documents, Wikipedia), computational models of operator learning, and extension to non-Indo-European families (Afro-Asiatic, Niger-Congo, Austronesian).

6. Conclusion

We document a striking dissociation in semantic space organization: semantic operators profoundly reorganize local topology while leaving global curvature largely unchanged. When operators (negators, modals, comparatives, intensifiers) are retained, they form coherent functional neighborhoods. When filtered using standard stopword lists, these neighborhoods dissolve. Yet global non-Euclidean structure (warp, SIR) remains stable across conditions. This pattern replicates across eight corpora spanning four languages and three language families, encompassing physics, biology, law, drama, sociology, and historical domains.

The central finding: Semantic systems appear to possess separable local and global organizational regimes. Operators structure immediate neighborhoods without requiring global space distortion. Different geometric mechanisms may govern functional relationships (local) versus conceptual hierarchies (global).

For natural language processing, the implication is clear: stopword filtering destroys local functional structure while appearing to preserve global geometric properties. The hidden cost is loss of operational semantics.

For cognitive science, the implications are deeper: if simple co-occurrence statistics can generate functionally organized operator neighborhoods, this suggests distributional learning may extract operational meaning (negation, modality, comparison) without supervision. Whether and how Hebbian mechanisms achieve this remains an open theoretical question.

The present work establishes the empirical phenomenon and reveals the local/global architectural separation. The theoretical mechanism—how associative learning extracts what “different” means from pure co-occurrence—awaits mechanistic investigation.

Figures

Figure 1. Operator Neighborhood Reorganization

Comparison of semantic neighborhoods for "much" in the Feynman Physics corpus under two conditions, visualized via PCA projection of high-dimensional coordinates. Left panel (ORIGINAL): Standard stopword filtering removes operators, resulting in dispersed neighbors (even, really, mean, believe, feel at 51–53 units) with no functional coherence. Right panel (OPERATOR_RESTORED): Operators retained, producing coherent functional cluster of evaluative/scalar terms (important, good, hard, easy, simple — blue), quantifiers (many — orange), intensifiers (really — purple), and epistemic verbs (knew — brown). Distances are substantially larger in OPERATOR_RESTORED (90–106 units vs. 51–53 units), reflecting stronger semantic differentiation when operators structure the space. This before/after pattern replicates across all eight corpora and four languages tested.

Figure 1. Operator Neighborhood Reorganization: "much" in Feynman Physics Corpus

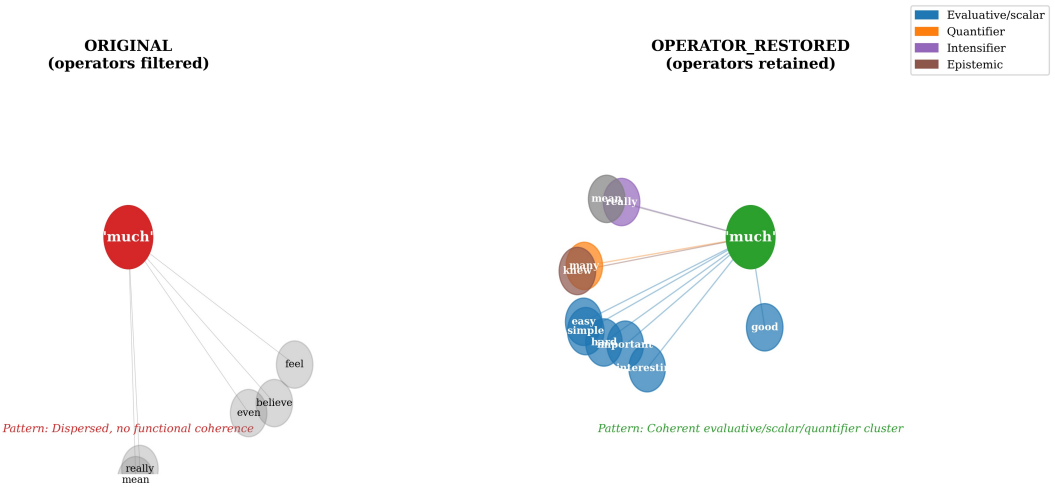

Figure 2. Global Geometry Varies by Domain, Not Language

Non-Euclidean structure measured by Warp Factor (top panel) and Spectral Imbalance Ratio (SIR, bottom panel) across eight corpora. Both measures show domain-driven variation: narrative texts (Shakespeare, Goethe) show lowest values; taxonomic texts (Darwin) show highest values; argumentative texts (legal, sociological, historical) cluster in middle range. Cross-language domain similarity exceeds within-language domain differences, suggesting conceptual organization dominates linguistic structure in determining global geometry.

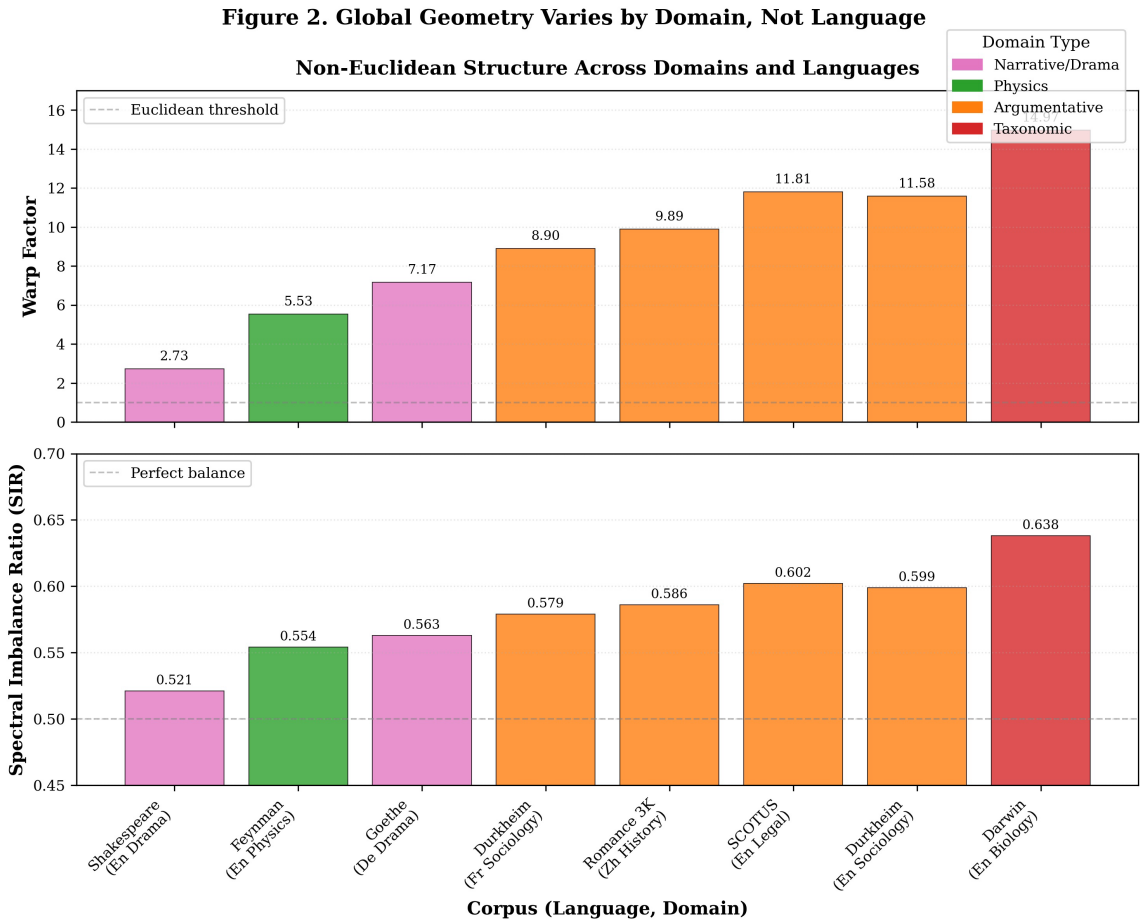

Figure 3. Local Reorganization Without Global Distortion

Schematic illustration of the local vs. global dissociation. Left: ORIGINAL condition shows random operator neighborhoods within semantic space (circle representing global structure). Right: OPERATOR_RESTORED condition shows coherent functional clusters (negators, modals, scalars) within nearly identical global space. Key finding: local topology reorganizes dramatically while global curvature remains stable (minimal changes in Warp and SIR).

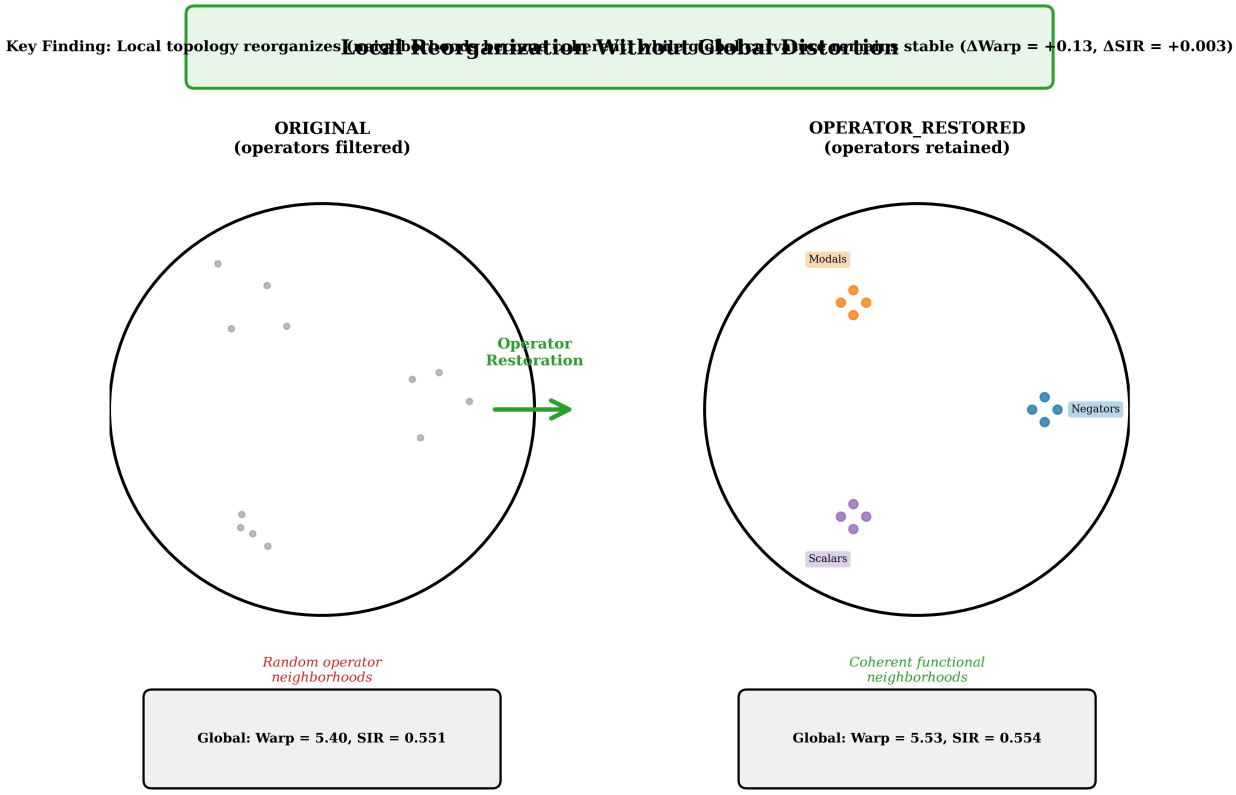

Figure 4. Cross-Language Operator Geometry Replication

Summary table showing consistent operator neighborhood patterns across four languages and three language families (Germanic, Romance, Sino-Tibetan). In all cases, ORIGINAL condition filters operators or produces random neighbors; OPERATOR_RESTORED condition produces functionally coherent clusters. Pattern strength ranges from moderate (Chinese 多) to very high (Chinese negators and modals, all Germanic/Romance operators), confirming cross-linguistic stability of operator geometry.

Figure 4. Cross-Language Operator Geometry Replication

Language	Family	Test Operator	ORIGINAL	OPERATOR_RESTORED	Coherence
English (5 corpora)	Germanic	"much"	Random neighbors	Scalar/comparative cluster	□ High
French	Romance	"beaucoup"	Filtered out	Temporal/scalar/ adverbial	□ High
German	Germanic	"viel"	Filtered out	Scalar/ descriptive	□ High
Chinese	Sino-Tibetan	"□" (not)	Retained (high freq)	Negation cluster	□ Very High
Chinese	Sino-Tibetan	"□" (can)	Retained (high freq)	Modal/epistemic cluster	□ Very High

Pattern confirmed across 3 language families (Germanic, Romance, Sino-Tibetan). Operators form functionally coherent neighborhoods when retained, random or filtered neighborhoods when removed.

References

Barnett, G. A., Woelfel, J., & Richards, W. D. (1993). A Galilean Model of Communication. In W. Richards & G. Barnett (Eds.), Progress in Communication Sciences (Vol. 12, pp. 1–41). Ablex.

Dhingra, B., Shallue, C., Norouzi, M., Dai, A., & Dahl, G. (2018). Embedding Text in Hyperbolic Spaces. Proceedings of TextGraphs-12, 59–69.

Frege, G. (1879). Begriffsschrift. Halle: Louis Nebert.

Gower, J. C. (1966). Some distance properties of latent root and vector methods used in multivariate analysis. Biometrika, 53(3–4), 325–338.

Kehler, A. (2002). Coherence, Reference, and the Theory of Grammar. CSLI Publications.

Kennedy, C. (2007). Vagueness and grammar: The semantics of relative and absolute gradable adjectives. Linguistics and Philosophy, 30(1), 1–45.

Kratzer, A. (1991). Modality. In A. von Stechow & D. Wunderlich (Eds.), Semantics: An International Handbook of Contemporary Research (pp. 639–650). De Gruyter.

Kripke, S. (1963). Semantical considerations on modal logic. Acta Philosophica Fennica, 16, 83–94.

Landauer, T. K., & Dumais, S. T. (1997). A solution to Plato's problem: The latent semantic analysis theory of acquisition, induction, and representation of knowledge. Psychological Review, 104(2), 211–240.

Lo, R. T., He, B., & Ounis, I. (2005). Automatically building a stopword list for an information retrieval system. Journal on Digital Information Management, 3(1), 3–8.

Mann, W. C., & Thompson, S. A. (1988). Rhetorical structure theory: Toward a functional theory of text organization. Text, 8(3), 243–281.

Manning, C. D., Raghavan, P., & Schütze, H. (2008). Introduction to Information Retrieval. Cambridge University Press.

Mikolov, T., Sutskever, I., Chen, K., Corrado, G. S., & Dean, J. (2013). Distributed representations of words and phrases and their compositionality. Advances in Neural Information Processing Systems, 26, 3111–3119.

Nickel, M., & Kiela, D. (2017). Poincaré embeddings for learning hierarchical representations. Advances in Neural Information Processing Systems, 30, 6338–6347.

Osgood, C. E., Suci, G. J., & Tannenbaum, P. H. (1957). The Measurement of Meaning. University of Illinois Press.

Pennington, J., Socher, R., & Manning, C. D. (2014). GloVe: Global vectors for word representation. Proceedings of EMNLP, 1532–1543.

Polanyi, L., & Zaenen, A. (2006). Contextual valence shifters. In J. G. Shanahan, Y. Qu, & J. Wiebe (Eds.), Computing Attitude and Affect in Text (pp. 1–10). Springer.

Pradana, M. S., & Hayaty, M. (2019). The effect of stemming and removal of stopwords on the accuracy of sentiment analysis on Indonesian-language texts. Kinetik, 4(3), 151–160.

Russell, B., & Whitehead, A. N. (1910). Principia Mathematica. Cambridge University Press.

Saif, H., Fernandez, M., He, Y., & Alani, H. (2014). On stopwords, filtering and data sparsity for sentiment analysis of Twitter. Proceedings of LREC, 810–817.

Salton, G., & McGill, M. J. (1983). Introduction to Modern Information Retrieval. McGraw-Hill.

Sparck Jones, K. (1972). A statistical interpretation of term specificity and its application in retrieval. Journal of Documentation, 28(1), 11–21.

Taboada, M., Brooke, J., Tofiloski, M., Voll, K., & Stede, M. (2011). Lexicon-based methods for sentiment analysis. Computational Linguistics, 37(2), 267–307.

Torgerson, W. S. (1952). Multidimensional scaling: I. Theory and method. Psychometrika, 17(4), 401–419.

von Stechow, A. (1984). Comparing semantic theories of comparison. Journal of Semantics, 3(1–2), 1–77.

Wiegand, M., Balahur, A., Roth, B., Klakow, D., & Montoyo, A. (2010). A survey on the role of negation in sentiment analysis. Proceedings of NeSp-NLP, 60–68.

Woelfel, J., & Fink, E. L. (1980). The Measurement of Communication Processes: Galileo Theory and Method. Academic Press.

Appendix A: Operator Lists by Language

English Operators

- **Negators**: not, never, no, nor, neither, nobody, nothing, nowhere, none
- **Intensifiers/Scalars**: very, much, too, extremely, highly, greatly, quite, rather, somewhat, fairly, pretty, really
- **Comparatives**: more, less, most, least, like, unlike, similar, different, same, than, as
- **Contrastives**: but, however, yet, although, though, nevertheless, nonetheless, despite, still
- **Modals**: can, could, may, might, must, should, would, shall, will, ought
- **Causals**: because, since, therefore, thus, hence, consequently, so, then, if, when

French Operators

- **Negators**: pas, ne, non, jamais, ni, rien, aucun, nul, personne
- **Intensifiers/Scalars**: très, beaucoup, trop, assez, plus, moins, fort, bien, mal

- **Comparatives**: plus, moins, comme, tel, pareil, différent, même, semblable
- **Contrastives**: mais, cependant, pourtant, toutefois, néanmoins, or, bien que
- **Modals**: peut, pourrait, doit, devrait, faut, saurait, voudrait
- **Causals**: parce, car, donc, ainsi, alors, si, puisque, comme

German Operators

- **Negators**: nicht, nie, kein, keine, nichts, niemand, niemals, nirgends
- **Intensifiers/Scalars**: sehr, viel, zu, mehr, wenig, genug, ganz, recht, ziemlich
- **Comparatives**: mehr, weniger, wie, als, gleich, ähnlich, verschieden, selbe
- **Contrastives**: aber, doch, jedoch, dennoch, sondern, trotzdem, obwohl
- **Modals**: kann, könnte, muss, sollte, mag, möchte, darf, würde, soll
- **Causals**: weil, denn, darum, deshalb, daher, also, wenn, dann, so, da

Chinese Operators (Classical)

- **Negators**: 不, 無, 沒, 未, 非, 莫, 毋, 勿
- **Intensifiers/Scalars**: 很, 太, 甚, 最, 更, 極, 頗, 殊
- **Comparatives**: 如, 若, 似, 比, 較, 同, 異, 猶
- **Contrastives**: 但, 然, 卻, 而, 只, 僅, 唯, 惟
- **Modals**: 可, 能, 應, 當, 該, 須, 必, 要, 宜
- **Causals**: 因, 故, 以, 為, 所, 由, 緣, 由於

Appendix B: Non-Euclidean Structure Metrics

Non-Euclidean curvature in semantic space is quantified using complementary measures derived from eigendecomposition of the double-centered matrix B = V Λ V^T.

Define the following quantities:

```
λ_pos = {λ_i | λ_i > 0}     (positive eigenvalues)
λ_neg = {λ_i | λ_i < 0}     (negative eigenvalues)

Σ_pos = Σ λ_pos             (sum of positive eigenvalues)
Σ_neg = Σ λ_neg             (algebraic sum, negative value)
Σ_abs = Σ |λ|               (sum of absolute values)
```

Warp Factor (historical measure, Woelfel & Fink, 1980):

```
Warp = Σ_pos / (Σ_pos + Σ_neg)
```

- Warp = 1.0: Euclidean (no negative eigenvalues)
- Warp > 1.0: Non-Euclidean (negative eigenvalues present)
- Unbounded; sensitive to small denominators

- Used for continuity with 40 years of Galileo research

Spectral Imbalance Ratio (SIR) (bounded alternative):

```
SIR = Σ_pos / Σ_abs
```

- SIR = 0.5: Perfect balance (equal positive and negative spectral energy)
- SIR = 1.0: All positive eigenvalues (Euclidean embedding)
- Bounded [0.5, 1.0]; interpretable as proportion
- Relationship: SIR = Warp / (2×Warp − 1)

Example (Feynman ORIGINAL):

```
Σ_pos = 23,887
Σ_neg = −19,458
Σ_abs = 43,345
Σ_all (algebraic) = 4,429

Warp = 23,887 / 4,429 = 5.40
SIR  = 23,887 / 43,345 = 0.551
```

Warp = 5.40 indicates positive eigenvalues dominate the algebraic sum by 5.4×. SIR = 0.551 indicates positive eigenvalues contain 55.1% of total spectral energy. Both measures indicate moderate non-Euclidean structure with positive eigenvalue dominance.

END OF DRAFT 1 — May 8, 2026